What Is the True Cost of Active Management? *A Comparison of Hedge Funds and Mutual Funds*

JUSSI KEPPO AND ANTTI PETAJISTO

JUSSI KEPPO
is associate professor in NUS Business School and Risk Management Institute at National University of Singapore.
keppo@nus.edu.sg

ANTTI PETAJISTO
is a portfolio manager at BlackRock.
antti.petajisto@sloan.mit.edu

Are hedge funds more expensive than mutual funds? Initially the answer may seem obvious: hedge funds typically charge a management fee of about 2%, slightly higher than the usual fee of about 1% charged by actively managed mutual funds, plus an incentive fee that typically amounts to 20% of any positive profits. Hence, hedge funds clearly charge higher fees per dollar invested.

Do they also deliver more in return for those fees? Naturally a hedge fund manager would claim they deliver greater alpha due to their superior skills. But what if the hedge fund and the mutual fund manager are about equally skilled? How do the mutual fund and hedge fund structures differ as vehicles to deliver alpha to investors, and which allows the investor to benefit more from a given manager's skills?

This question has become more important recently as new legislation and regulation in the United States and the European Union has been increasing the costs of opening new hedge funds, thus making the mutual fund structure relatively more attractive to active managers. To the extent that this regulatory change pushes active managers to choose to run mutual funds instead of hedge funds, does this mean investors are getting a better deal and paying lower fees for active management? We point out that the answer is far from obvious.

In this article we investigate whether investors are better off investing in hedge funds or mutual funds, and how this trade-off varies for a range of plausible parameter values. We also explain the costs and benefits of each fund structure from the investor's point of view. Our approach is conceptual: We want to understand the mechanism of the trade-off, so rather than estimating various relevant quantities from the hedge fund and mutual fund data ourselves, we use other researchers' estimates to evaluate the trade-off for investors. Further, by using the trade-off, our model explains several documented empirical findings on the career development of successful fund managers and on hedge funds' risk-taking (see e.g., Li et al. [2011] and Nohel et al. [2010]).

The simplest trade-off comes from the effect of leverage: A hedge fund can use leverage to scale up the manager's bets for each dollar of the investor's capital, generating a greater alpha, albeit with greater idiosyncratic volatility, than the active mutual fund, and hence offsetting the higher management fee. In this sense, the hedge fund can indeed provide more active management for each dollar invested.

Another potential cost for mutual-fund investors arises from the market exposure embedded in their actively managed fund: If investors are not able (perhaps due to institutional constraints) to hedge out the market exposure themselves, they may end up with too much beta and too little alpha. In contrast, a market-neutral hedge fund gives them an easy way to optimize their exposure to both the market and the active strategies. Other smaller effects also can affect the trade-off; for example, if the hedge fund is able to borrow at lower rates or borrow more than the investor himself, that can add value if the investor would like to use some leverage to boost his returns.

According to our model, assuming a moderately skilled equity manager who has an annualized information ratio of about 0.3 before fees, mutual funds and hedge funds deliver about the same expected utility to investors, so both investment vehicles are equally attractive, in spite of the fact that hedge funds may initially appear to be more expensive. Investors benefit from hedge fund leverage, since the management fee is paid on the money invested in the fund, not on the gross positions that include leverage. This leverage effect is stronger the higher the hedge fund's information ratio before fees (or "skill," loosely speaking). Further, if the fund is more levered, the investor can effectively get the same exposure with a smaller investment in the fund. We find that without leverage typical hedge funds could not compete with mutual funds, and that these findings are quite robust with respect to a jump risk in the hedge fund returns. We also investigate impacts from several other factors. For instance, our model shows that a 10-percentage-point increase in the incentive fee should be compensated by a little more than a 1-percentage-point increase in the unlevered alpha.

Several papers have empirically analyzed hedge-fund alphas and fees. Early studies of hedge-fund performance include, for example, Fung and Hsieh [1997a, 1997b, 1999, 2000, 2001]; Ackermann et al. [1999]; Brown et al. [1999, 2000, 2001]; Liang [1999, 2000, 2001]; Agarwal and Naik [2000a, 2000b, 2000c]; Brown and Goetzmann [2001]; Edwards and Caglayan [2001]; Kao [2002]; and Lochoff [2002]. Getmansky et al. [2004] and Khandani and Lo [2009] point out that autocorrelation in returns induced by return smoothing may distort performance measures of some hedge funds. Joenvaara [2011] and Bali et al. [2011] argue that hedge fund alphas in good times are in part compensation for systemic risk, while Jylha and Suominen [2011] attribute part of hedge fund alphas to a simple carry trade strategy. Fung et al. [2008] find that while some hedge funds appear to generate true alphas, inflows to the best-performing funds and the hedge fund industry overall appear to have pushed these alphas down. Kritzman [2008] provides an example of fees and returns for a hedge fund and mutual fund, demonstrating the importance of considering the size of active bets in this comparison. In general, while most studies find a positive level of skill for hedge funds, quantifying hedge fund performance remains difficult because of the wide variety of investment styles across funds, time-varying strategies within funds, and lack of comprehensive data due to voluntary reporting, survivorship bias, and backfill bias.

In contrast, mutual fund performance has been studied over a long time period and using comprehensive data: For example, Jensen [1968]; Brown and Goetzmann [1995]; Carhart [1997]; Grinblatt and Titman [1989, 1993]; Gruber [1996]; Daniel et al. [1997]; Wermers [2000, 2003]; Pastor and Stambaugh [2002]; Bollen and Busse [2004]; Cohen et al. [2005]; and Mamaysky et al. [2007]. While the average fund has lost to its benchmark net of fees and expenses, most papers find positive before-fee alphas of about 1%, indicating some positive average level of skill. More recently the literature has focused on identifying subsets of mutual fund managers that are more likely to outperform. For example, Cremers and Petajisto [2009] and Petajisto [2010] introduce Active Share as a way to quantify how active fund managers are, pointing out that the most active stock pickers on average have been able to outperform their benchmarks even after fees and transaction costs. Such evidence is reassuring for our analysis, which starts with the premise that some investors can indeed identify value-adding managers. Even if one were to disagree with this premise, the fact remains that trillions of dollars are invested with active managers, so improving this decision can significantly improve the welfare of investors.

The rest of the article is organized as follows. The next two sections present our simple model and our optimization procedure. The article then proceeds to analyze and discuss the calibration results. The main conclusions are summarized in the final section.

MODEL

We consider an investor who is able to invest in an index fund, active mutual fund, and hedge fund, as well as borrow and lend cash. The investor can borrow and lend at the same risk-free rate and use leverage up to $L \geq 0$ times his wealth. His investment strategy is static over time horizon $[0, T]$, so at time 0 the investor selects the portfolio and then just passively waits until the payoffs are realized at time T. However, if at any time between 0 and T the portfolio value falls below a certain threshold, the institution that loaned the money will ask the investor to close his entire risky investment. The investor's objective is to maximize expected utility from his wealth at time T, given constant relative risk aversion (CRRA) preferences.

The value of the risk-free investment at time T is given by

$$W_r(T) = W_r(0)\ \exp(rT) \tag{1}$$

where r is the risk-free rate and $W_r(0)$ is the amount of money in the risk-free asset at time 0. As discussed previously, the investor is able to borrow and lend, that is, he can take long and short positions in the risk-free asset. However, $W_r(0) \geq -LW(0)$, where $W(0) > 0$ is the initial total wealth of the investor and $L \geq 0$ is the maximum leverage level; that is, the maximum loan the investor can take is L times the initial wealth.

The investor can also invest in an index fund that holds the market portfolio. The value of the index fund follows a geometric Brownian motion, and thus the value at time T is given by

$$W_i(T) = W_i(0)\exp\left(\left(r + \eta\sigma_i - \frac{1}{2}\sigma_i^2\right)T + \sigma_i B_i(T)\right) \tag{2}$$

here $W_i(0)$ is the amount of money in the fund at time 0, $B_i(t)$ is a standard Brownian motion, η is the market price of risk (or Sharpe ratio) corresponding to $B_i(t)$, and σ_i is the index volatility. For simplicity, the index fund does not charge any fees.

The active mutual fund also follows a geometric Brownian motion process and the value of the fund after fees at time T is given by

$$\begin{aligned} W_a(T) = {} & \left[W_a(0)(1-f_a)^T\right] \\ & \times \exp\left\{\left[r + \eta\sigma_{ai} + \sigma_a - \frac{1}{2}\left(\sigma_{ai}^2 + \sigma_{aa}^2\right)\right]T \right. \\ & \left. + \sigma_{ai}B_i(T) + \sigma_{aa}B_a(T)\right\} \end{aligned} \tag{3}$$

where $f_a \in (0, 1)$ is the annual management fee (charged over T years), $W_a(0)$ is the amount of money (before the fee) invested in the fund at time 0, $B_a(t)$ is a standard Wiener process independent of $B_i(t)$, σ_{ai} and σ_{aa} are the volatility coefficients corresponding to $B_i(t)$ and $B_a(t)$, and α_a is the before-fee alpha of the active fund. The correlation of returns between the fund and the market index is $\sigma_{ai}/\sqrt{\sigma_{ai}^2 + \sigma_{aa}^2}$. Consistent with, for example, the CAPM, we assume that $B_a(t)$ has zero market price of risk because it is independent of the market (index fund), and therefore the expected return according to the CAPM (i.e., when alpha is zero) is $r + \eta\sigma_{ai}$. Mutual funds face constraints on their leverage due to the Investment Company Act of 1940, and their maximum leverage varies from zero to 33%. However, in practice most mutual funds do not use any leverage at all (see, e.g., Almazan et al. [2004]). We do not model the mutual fund leverage explicitly since it is at a low level and, therefore, the value dynamics in Equation (3) are directly under the fund's leverage. The management fee f_a is a percentage of assets invested in the mutual fund, so as seen in Equation (3), $W_a(0)(1 - f_a)^{\mathrm{T}}$ is the wealth invested in the fund after the fee.

The value of the hedge fund before the incentive fee also follows a geometric Brownian motion. The hedge fund uses leverage, that is, it borrows π times the money invested in the fund. The borrowing rate equals the risk-free rate r and leverage is constant between $t = 0$ and $t = T$. The hedge fund charges an annual management fee f_h. The value of the investor's wealth in the hedge fund after the management fee is then given by

$$\begin{aligned} W_h(T) = {} & \left[W_h(0)(1 - f_h)^T\right] \\ & \times \exp\left\{\left[r + (1+\pi)(\eta\sigma_{hi} + \alpha_h) - \frac{1}{2}(1+\pi)^2 \right.\right. \\ & \left.\left. \times (\sigma_{hi}^2 + \sigma_{hh}^2)\right] T + (1+\pi)\left[\sigma_{hi}B_i(T) + \sigma_{hh}B_h(T)\right]\right\} \end{aligned} \tag{4}$$

where $W_h(0)$ is the amount of money (before the fee) invested in the fund at time 0, $B_h(t)$ is a standard Brownian motion independent of $B_i(t)$, σ_{hi} and σ_{hh} are the volatility coefficients corresponding to $B_i(t)$ and $B_h(t)$, and α_h is the before-fee alpha of the hedge fund strategy. As with the mutual fund, $B_h(t)$ is independent of the market (index fund) and its market price of risk is assumed to be zero. In addition to the management fee, at time T a performance-based incentive fee $f_p \in [0, 1)$ is charged as a percentage of any positive profits. Thus, at time T the after-fee value of the hedge fund is

$$W_h(T) - f_p \max[W_h(T) - W_h(0), 0] \tag{5}$$

The performance fee is similar to a call option, so Equation (5) indicates that the investor gives the hedge fund manager f_p call options on the fund value (after the management fee has been subtracted) with a strike price of $W_h(0)$ and maturity T. Thus, the investor is long the fund and short call options on the fund.

Our model is stylized and we ignore several features in the fund business as they do not play a first-order role in the trade-offs that we want to investigate. These include the following:

- We model just one period (e.g., static asset allocation for one year) and thus we ignore any long-term dynamics. For example, many hedge funds have a high-water mark, which means that if they lose money in a given year, then in future years they first have to recoup their losses in full before they can charge incentive fees to investors. While this may sound like a great deal to hedge fund investors, sometimes hedge fund managers shut down their funds after they suffer serious losses and start new ones, thus resetting their high-water marks rather than attempting to recover the earlier losses over a number of years without incentive fee.[1] Because the possibility of a manager restart offsets the benefits of the high-water mark, it is not clear what the net effect is for hedge fund investors.
- We ignore taxes. This also could potentially favor either the hedge fund or mutual fund. Mutual fund investors may pay higher taxes if inflows and outflows due to other investors generate additional portfolio turnover and higher realized capital gains each year. Hedge fund investors in turn have to be careful of more convoluted practices, such as "stuffing," which means that the fund uses its discretion to allocate short-term capital gains to the redeeming investors and long-term capital gains to the remaining investors (and the general partner), even though everyone of course had true economic exposure to the same underlying portfolio.
- We assume that the funds allow investors to withdraw their money in cash at the end of the period, or if a fund hits its liquidation boundary before that (for discussion on this see, e.g., Ineichen [2002]). In reality, a hedge fund may put up "gates" to stop such withdrawals, which is usually justified by arguing that orderly withdrawals (rather than mass withdrawals, especially from illiquid investments) are in the interests of the hedge fund investors themselves.
- We ignore hidden fees and expenses such as trading commissions (including soft dollars) and price impact. We also ignore index fund fees. Index funds for U.S. large-cap indexes typically charge 0.06%–0.20% annually. Furthermore, some hedge funds charge a variety of other expenses directly to the fund, in addition to their typical annual management fee of about 2%.

Nevertheless, we believe that our model captures the main fees and variables that affect the trade- offs from the perspective of a fund investor choosing between a mutual fund and a hedge fund.

OPTIMIZATION

The investor invests in the risk-free asset, the index fund, the active mutual fund, and the hedge fund. If the investor uses leverage, that is, if $W_r(0) < 0$, then the institution that loaned the money continuously monitors the investor's wealth and closes his risky positions immediately if his total wealth hits $\varepsilon W(0)$, where $\varepsilon \in (0, 1)$ and $W(0)$ is the initial total wealth.[2] After the liquidation of all risky positions, the investor has $\varepsilon W(0)$ in the risk-free asset. We denote this liquidation time as τ and, formally, it is given by

$$\tau = \inf\{t : W(t) \leq \varepsilon W(0)\}$$

where at time $t \leq \tau$, the investor's total wealth is

$$\begin{aligned} W(t) = {} & W_r(t) + W_i(t) + W_a(t) + W_h(t) \\ & - f_p \max[W_h(t) - W_h(0), 0] \end{aligned}$$

By (1)–(5) and the discussion above, we can write the investor's total wealth at time T as

$$\begin{aligned} W(T) = {} & (1 - I\{\tau < T\} I\{W_r(0) < 0\}) \\ & \times \{W_r(T) + W_i(T) + W_a(T) + W_h(T) \\ & \quad - f_p \max[W_h(T) - W_h(0), 0]\} \\ & + I\{\tau < T\} I\{W_r(0) < 0\}\, \varepsilon W(0) \\ & \times \exp(r(T - \tau)) \end{aligned} \tag{6}$$

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such that $W(0) = W_r(0) + W_i(0) + W_a(0) + W_h(0)$, where $W(0)$ is the initial total wealth and the indicator function $I\{A\} = 1$ if A is true and otherwise it is zero. Thus, there are two cases: No liquidation, represented by the first three lines of (6), and liquidation, represented by the last two lines of (6). The liquidation happens if the investor uses leverage ($W_r(0) < 0$) and if the portfolio value falls enough ($W(T) = \varepsilon W(0)$).

The investor follows a buy-and-hold strategy, deciding his allocation to different funds at time 0 and then just waiting (and not trading) until time T. Given the investor's CRRA preferences, his objective function is:

$$\max_{W_r(0) \geq -LW(0),\, W_i(0) \geq 0,\, W_a(0) \geq 0,\, W_h(0) \geq 0} E\left[\frac{W^{1-\gamma}(T)}{1-\gamma}\right] \quad (7)$$

such that the wealth dynamics are given by (1)–(6), and the relative risk aversion coefficient is $\gamma > 0$, $\gamma \neq 1$. Hence, the investor is not able to short sell the funds, but short selling of the risk-free asset (i.e., borrowing cash) is allowed up to L times the initial wealth. Note that since the investor can always choose to invest zero in any risky asset, he can only benefit from having more options in which to invest. We solve the optimization problem by Monte Carlo simulation with 50,000 simulation runs with 25-trading-day time increments and by the method of Lagrange multipliers (see, e.g., Glasserman [2003] and Bazaraa et al. [1993]).[3] We use Monte Carlo simulation because we have to allow for the possibility that the portfolio is liquidated before T, which makes it more difficult to find a convenient analytical expression for the probability density function of terminal wealth in Equation (6).

Our objective is to see how much value either a hedge fund or an active mutual fund adds to investors, so we need to compare them independently of each other. Hence, we impose the additional constraint in Equation (7) that the investor's position in one of the two active funds has to be zero. Note that optimally an investor would allocate some of his wealth to all three funds: the index fund, active mutual fund, and hedge fund. However, our objective is not to answer this question, but rather to understand how much value a skilled active manager adds to the investor if he sets up a mutual fund or a hedge fund, and to see how the various parameters of each fund (leverage, tracking error, and so forth) affect this comparison between the two types of funds. Note that in this analysis we cannot replicate the payoff distribution of a mutual fund with a hedge fund because these investments have different time-T payoff probability distributions due to the incentive fee in Equation (5). This is the reason we use the set-up introduced in this section when comparing a hedge fund with a mutual fund.

CALIBRATION

Selection of Parameters

To investigate the economic significance of the trade-offs between a mutual fund and a hedge fund, we need to calibrate our model to realistic parameter values. Whenever the parameter estimates are hard to pin down, we explore the sensitivity of our results to a range of reasonable parameter values. For the market portfolio, we use a risk premium of 5% and a volatility of 20% (i.e., the market price of risk or annual Sharpe ratio is 0.25, which is close to the S&P 500's long-term annual Sharpe ratio). For the risk-free rate, we pick 3%, which implies a 0%–1% real interest rate as long as inflation stays in the 2%–3% range. Our time horizon is one year.

All-equity mutual funds have a market beta very close to 1, so their market volatility is 20%. For the idiosyncratic volatility of mutual funds, also known as tracking error, we select 6%, which is a rough average for U.S. equity mutual funds reported by Cremers and Petajisto [2009]. The mutual fund alpha is of course difficult to determine; we start with a 2% before-fee alpha, so that when combined with a 1% annual management fee, the investor still earns 1% per year and therefore has a rational reason to invest in an actively managed mutual fund. Empirical evidence shows that while the average actively managed mutual fund has some skill or before-fee alpha, about 1% per year, the net alpha to investors after all expenses is slightly negative (e.g., Wermers [2000] and Daniel et al. [1997]). This implies that randomly selected mutual funds should optimally receive a zero allocation because they are dominated by index funds. However, many investors believe they have the skill to select managers who actually add value net of fees and we start our analysis with those investors. Furthermore, empirical evidence shows that even simple rules, such as screening out funds with negative momentum (Carhart [1997]), large size (Chen et al. [2004]), low levels of active management (Cremers and Petajisto [2009]), or broker-sold retail funds (Del Guercio and Reuter [2014]), create subsets of active managers who earn on average higher returns than the mutual fund population overall.

For the unlevered volatility of the hedge fund strategy, we start with 10%, which is also consistent with both the empirical estimates and calibrations of Getmansky et al. [2004]. Our benchmark case is a market-neutral hedge fund, but we also allow the hedge fund to have exposure to market risk, varying its market beta from zero to one. Since hedge funds use leverage, their levered volatility tends to be higher; we vary leverage from zero to four, meaning that we go from unlevered returns to levered returns that are five times as large. The common value we pick for leverage is one, meaning the unlevered positions are scaled up by a factor of two. This is consistent with a levered volatility of 21.69% reported by Getmansky et al. [2004] for U.S. equity hedge funds, and the average hedge fund leverage reported in Ang et al. [2011].[4]

The unlevered hedge fund alpha is one of the key parameters, but it is also notoriously difficult to estimate. There is no comprehensive database for hedge fund returns; the managers engage in nonlinear and time-varying strategies; and some strategies involve illiquid assets, where the current market value is somewhat ambiguous. Hence, the hedge fund alpha is a free parameter in most of our calibrations. We start with 3% per year and vary the parameter from zero to 7% per year. Note that this implies an annual information ratio, defined as the expected active return divided by the tracking error (also called the "appraisal ratio" by Treynor and Black [1973]) of 3/10 = 0.3, while the mutual fund's information ratio is 2/6 = 0.33, which is essentially the same.[5] Thus, implicitly we assume that the hedge fund manager and the mutual fund manager are equally skilled at finding good investment opportunities.[6] In contrast to alphas, hedge fund fees are relatively transparent and usually equal to about 2% of assets per year and about 20% of any positive returns. More specifically, by Feng [2011], the median management fee and incentive fee are 1.5% and 20%, and we start with these estimates.

From the investor's objective function (7) and wealth dynamics (6), we see that the investor is defined by his relative risk aversion coefficient γ, planning horizon T, maximum leverage L, and liquidation trigger level ε. Consistent with the literature (see, e.g., Mehra and Prescott [1985] for early references), we select $\gamma = 2$. Combined with a 5% equity premium and a 20% equity market volatility (thus $\eta = 0.25$), this implies an optimal allocation of 62.5% of the investor's wealth to equities,[7] which is again consistent with most of the literature (see, e.g., Cocco et al. [2005]). We select an investment horizon equal to one year, that is, $T = 1$, and we assume that the investor is able to take the same amount of leverage as the hedge fund under the initial parameter values, that is, $L = 1$. We set $\varepsilon = 0.02$, which means that the portfolio is liquidated if the investor uses leverage and if the portfolio value falls 98% during the one-year period.

In summary, we have the following initial values for the model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\alpha_a = 0.02$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $f_p = 0.2$, $\pi = 1$, $\alpha_h = 0.03$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, $L = 1$, and $\varepsilon = 0.02$. In the next section, we vary many of these parameter values in order to analyze their effects on expected utility and optimal fund allocation.

Calibration Results

To compare the attractiveness of the hedge fund and the mutual fund, we compute the certainty equivalent return of each alternative to the investor. As discussed earlier, the investor chooses his optimal allocation to either 1) a hedge fund, index fund, and the risk-free asset, or 2) a mutual fund, index fund, and the risk-free asset. The certainty equivalent indicates how much an investor, who starts out holding everything in cash, would be willing to pay (in percent per year) for having those new investment options offered to him. It therefore has a convenient economic interpretation as the value those investment options create for the investor, expressed in terms of wealth and not units of utility.

The certainty equivalent return can be computed from the investor's value function:

$$r_{CE} = \frac{W_{CE}(T)}{W(0)} - 1 = \frac{[(1-\gamma)V]^{\frac{1}{1-\gamma}}}{W(0)} - 1 \qquad (8)$$

where Equation (7) allows us to substitute $V = E\left[\frac{W^{1-\gamma}(T)}{1-\gamma}\right] = \frac{W_{CE}^{1-\gamma}(T)}{1-\gamma}$, with $W(T)$ denoting terminal wealth under the optimal fund allocation. Alternatively, it can be computed from the investor's return distribution directly:

$$r_{CE} = E\left[(1+r_W)^{1-\gamma}\right] - 1 \qquad (9)$$

where r_W is the net (percentage) return on the investor's total wealth from $t = 0$ to $t = T$.

Hedge Fund Alpha versus Leverage. Exhibit 1 shows the investor's optimal investment in the hedge fund as a function of the unlevered hedge fund alpha, hedge fund leverage (Exhibit 1, Panel A), and the investor's own maximum leverage (Exhibit 1, Panel B). If the unlevered hedge fund alpha is 3% per year and the fund is unlevered, the investor invests about 16% of his wealth in the hedge fund, and he does not use any leverage himself (not directly reported in Exhibit 1). However, hedge fund leverage makes the fund more attractive, because the investor is not paying the management fee of 1.5% on the levered positions but only pays the incentive fee on them, so the investor more than doubles the hedge fund investment when the hedge fund leverage rises from zero to one (the investment increases from 16% to 35%).

At the same time, if the fund is more levered, the investor can effectively get the same exposure with a smaller investment in the fund; this competing effect makes the investor reduce his allocation, especially for higher levels of alpha. By Exhibit 1, Panel B, the investor has maximum leverage (L) when the hedge fund alpha is high. In this case, the investor wants to use leverage to scale up the manager's bets and the leverage constraint becomes binding. Note that, as discussed earlier, the investor would prefer the hedge fund to take the leverage: if the hedge fund uses higher leverage, it is more cost effective for the investor because then the management fee is lower.[8]

Exhibit 2 shows the difference in certainty equivalent between the hedge fund and the mutual fund, computed from Equation (8). This difference is shown as a contour plot where zero denotes indifference between the two options, and 0.02 indicates that the investor prefers the hedge fund with a margin of 2% of his wealth per year. The exhibit shows all contours for increments of 2%.

For an unlevered hedge fund with an alpha of 3%, the investor prefers the mutual fund, but he begins to prefer the hedge fund after its leverage reaches 100%. The indifference point is, perhaps surprisingly, very close to the initial parameter values we picked: an alpha of 3% and leverage of one. Note that we assumed a mutual fund alpha of 2% before fees with 6% idiosyncratic volatility, which implies an information ratio of 0.33 for the mutual fund, while the hedge fund was assumed to have an idiosyncratic volatility of 10%, which gives it an information ratio of 0.3. Hence, with these parameters we are effectively assuming the hedge fund

EXHIBIT 1
Optimal Wealth Allocated to Hedge Fund and Risk-Free Asset

Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\alpha_a = 0.02$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $f_p = 0.2$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, and $\varepsilon = 0.02$.

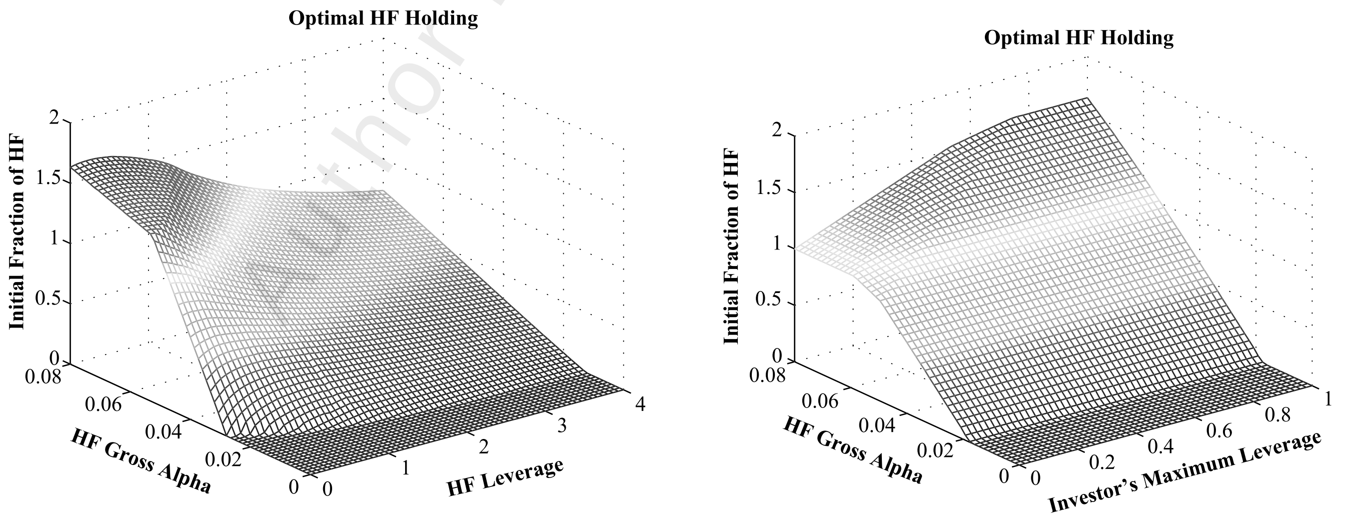
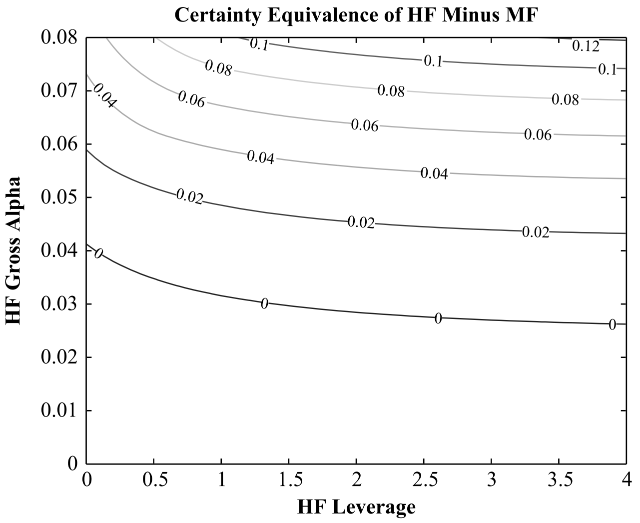


EXHIBIT 2

Difference in Certainty Equivalents of Hedge Fund and Mutual Fund as a Function of Hedge Fund Alpha (α_h) and Hedge Fund Leverage (π)

Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\alpha_a = 0.02$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $f_p = 0.2$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, $L = 1$, and $\varepsilon = 0.02$.



and mutual fund manager are almost equally good in finding investment opportunities, and the question then reduces to which of the two investment vehicles gives the investor more cost-effective access to the manager's expertise. The main economic conclusion here is that the two investment vehicles are equally attractive: the slightly greater unlevered exposure of the hedge fund, combined with its use of leverage, almost exactly offset the manager's incentive fee. Hence, given a moderate level of skill for an active manager, the choice of the fund structure does not matter, in spite of the seemingly higher fees of hedge funds.[9]

Li et al. [2011] study the impact of manager characteristics, such as education and career concern, on hedge fund performances. They find that hedge fund managers graduated from selective colleges tend to take fewer risks, have higher returns, and attract more capital inflows. These findings are consistent with Exhibits 1 and 2, which show that high-alpha hedge fund managers do not need leverage (which raises risk) to attract investors.

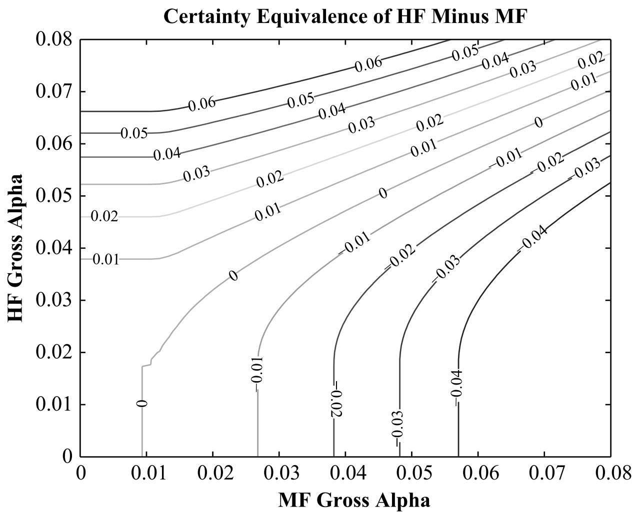
Hedge Fund Alpha versus Mutual Fund Alpha. Exhibit 3 explores how the investor's decision depends on the alpha of the hedge fund and the alpha of the mutual fund. In order to add value, each fund has to offer an alpha that exceeds its total fees; for the mutual fund this is simply 1%, and for the hedge fund it requires about 2% unlevered alpha (with a leverage of one). But if we then increase the alpha of each manager at the same rate, we find that the investor begins to prefer the hedge fund: the slope of the indifference curve (i.e., the curve where the difference between the two certainty equivalents is zero) is only about one-half. This means that high-alpha managers are offering more value to their investors if they work at a hedge fund rather than a mutual fund. This is consistent with Nohel et al. [2010], who report that several star mutual fund managers simultaneously manage mutual funds and hedge funds.

If instead we increase the information ratio of each manager at the same rate, then the preference for hedge funds becomes even stronger. Note that in our current calibration, the hedge fund has a higher idiosyncratic volatility than the mutual fund, so assuming the same alpha for the two is equivalent to assuming a higher information ratio (i.e., more skill) for the mutual fund manager. Hence, Exhibit 3 may appear to understate the advantage of the hedge fund structure.

EXHIBIT 3

Difference in Certainty Equivalents of Hedge Fund and Mutual Fund as a Function of Hedge Fund Alpha (α_h) and Mutual Fund Alpha (α_a)

Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $f_p = 0.2$, $\pi = 1$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, $L = 1$, and $\varepsilon = 0.02$.



The investor prefers high-alpha managers in hedge funds mostly because the mutual fund is a package deal: taking a large position in the active bets of the manager also requires the investor to take a large bet on the passive market index. In other words, the mutual fund offers too little exposure to the active bet and too much exposure to the market index. If the investor can short the market index, he can get around this problem by eliminating the excessive index position from his portfolio, but in reality a large class of investors cannot take short positions in the index and thus are subject to this problem with the mutual fund.[10] The higher the alpha of the manager, the more binding the constraint. In contrast, the hedge fund offers a pure bet on the active strategy, which is easy to optimally combine with a position in the market index.

A smaller effect benefiting the hedge fund with higher levels of alpha arises from the increasing symmetry of the incentive fee: the more likely it is that the fund is in positive territory, the more symmetric the incentive fee will be. For example, assume the expected hedge fund return is 10% before fees, which is 8% after the 20% incentive fee. If the fund's realized gross return is 20%, this translates to a 16% net return; if its realized gross return is zero, this translates to a zero net return. In other words, the incentive fee is completely symmetric in this range: it charges the investor 20% of unexpected positive shocks but it also effectively refunds him 20% of unexpected negative shocks. The biggest cost of the incentive fee to the investor is therefore its option value, and that option value is diminished the deeper in the money the option is.

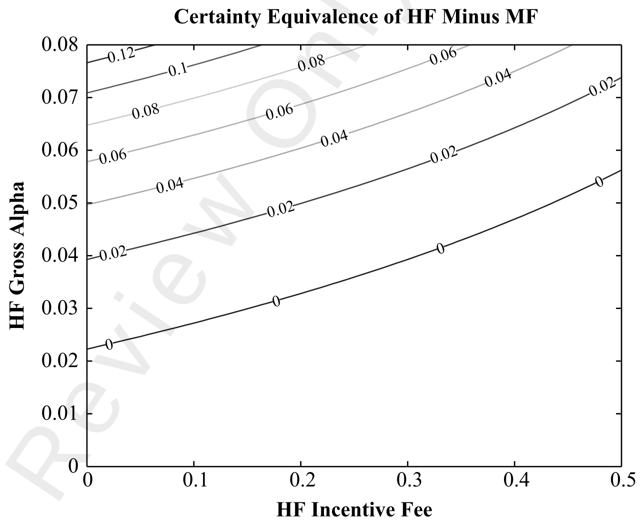
Hedge Fund Alpha versus Incentive Fee. Not surprisingly, the incentive fee has a direct and significant impact on the trade-off between the hedge fund and mutual fund, as shown in Exhibit 4. With a 20% incentive fee and an unlevered hedge fund alpha of 3%, the investor is indifferent between the hedge fund and mutual fund. If the incentive fee is increased to 40%, the investor requires over 4% of unlevered alpha to remain indifferent between the two funds. Hence, an additional 20 percentage points in the incentive fee eats up slightly more than 1 percentage point of unlevered alpha. In reality some funds even charge incentive fees of 50%, which naturally requires very high alphas to become attractive to investors.

The fund fees naturally affect portfolio managers' behavior. For instance, Li et al. [2011] find that, since

EXHIBIT 4

Difference in Certainty Equivalents of Hedge Fund and Mutual Fund as a Function of Hedge Fund Alpha (α_h) and Hedge Fund Incentive Fee (f_p)

Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\alpha_a = 0.02$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $\pi = 1$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, $L = 1$, and $\varepsilon = 0.02$.



a significant part of hedge fund compensation comes from incentive fees, hedge fund managers may not want to grow their funds to the extent that all excess returns disappear.

Hedge Fund Volatility versus Leverage. Leverage varies nontrivially across equity hedge funds, and while voluntarily reported data are available, we still wish to investigate the impact of leverage for a range of plausible values, as shown in Exhibit 5. Keeping the alpha of the hedge fund constant, a lower volatility for the hedge fund has the effect of making it significantly more attractive. This naturally arises from the increase in information ratio. This effect is particularly dramatic for higher levels of hedge fund leverage, because leverage allows the investor to scale a high information ratio to a return that also has a high absolute level of alpha, and leverage also mitigates the negative impact of the management fee. These findings are consistent with Li et al. [2011], who find that hedge fund managers who take less risks tend to attract more capital inflows.

Hedge Fund Alpha Versus Beta. Exhibit 6 shows what happens when the hedge fund starts to bear exposure to the overall market and increases its market

EXHIBIT 5

Difference in Certainty Equivalents of Hedge Fund and Mutual Fund As a Function of Hedge Fund Volatility (σ_{hh}) and Hedge Fund Leverage (π)

Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\alpha_a = 0.02$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $f_p = 0.2$, $\alpha_h = 0.03$, $\sigma_{hi} = 0$, $\gamma = 2$, $T = 1$, $L = 1$, and $\varepsilon = 0.02$.

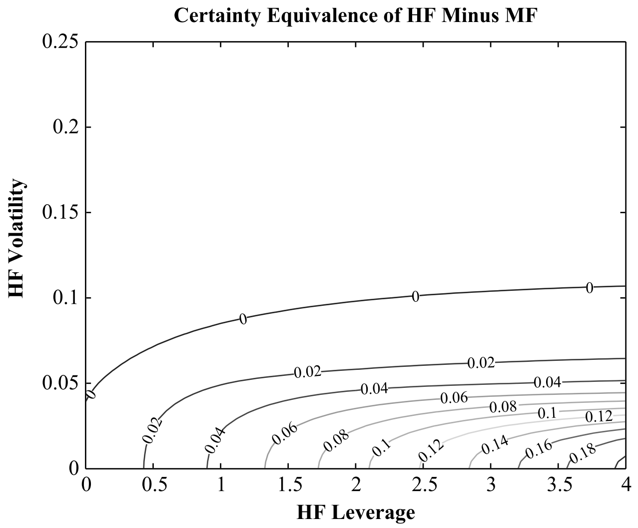
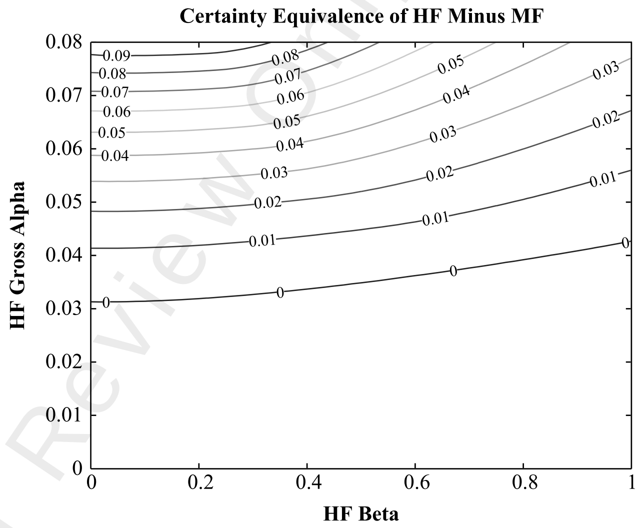


EXHIBIT 6

Difference in Certainty Equivalents of Hedge Fund and Mutual Fund As a Function of Hedge Fund Alpha (α_h) and Hedge Fund Beta (β)

Beta changes as a function of σ_{hi}: $\beta = \sigma_{hi}/\sigma_i$. Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\alpha_a = 0.02$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $f_p = 0.2$, $\pi = 1$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, $L = 1$, and $\varepsilon = 0.02$.



beta from zero to one. By Equation (4) and the capital asset pricing model, we have $\beta = \sigma_{hi}/\sigma_i$, where β is the capital asset pricing model's beta. That is, in Exhibit 6 beta changes due to the changes in σ_{hi}.

For the investor to remain indifferent between the hedge fund and the mutual fund as the hedge fund beta increases from zero to one, the hedge fund would need to increase its alpha by about 1% per year, meaning that its information ratio would have to increase by one third. A market beta of one essentially turns the hedge fund into a very expensive mutual fund—one that charges not only a high management fee but also an incentive fee, thus taking a fraction of profits even when they are entirely due to the overall market going up, and not due to the manager's astute active bets. Overall, hedge fund investors are much better off with a true market-neutral fund that offers them pure alpha exposure and nothing else.

Mutual Fund Idiosyncratic Volatility versus Mutual Fund Alpha. In Exhibit 7, we see how the trade-off depends on mutual fund parameters, in particular alpha and idiosyncratic volatility. Reducing idiosyncratic volatility helps the mutual fund, but not nearly as much as it would help the hedge fund. The reason is that the mutual fund will still have exposure to market risk, and our traditional long-only investor cannot isolate the mutual fund alpha from the mutual fund beta.

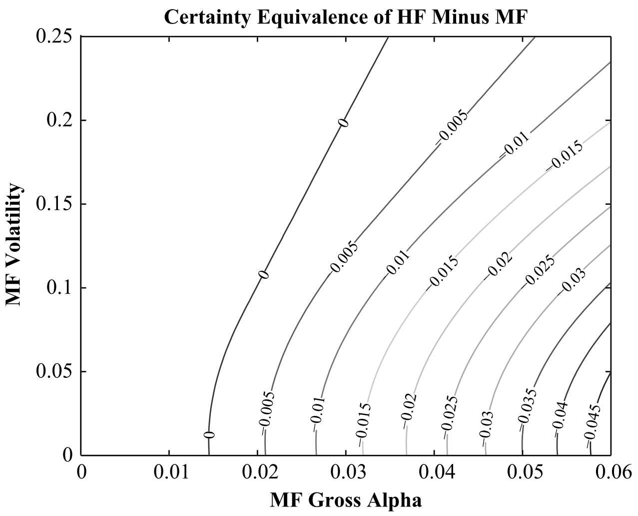
Skewness. To analyze the robustness of our results, in this subsection we consider a situation where the hedge fund value has negative skewness. This tail risk is documented, for example, in Kelly and Jiang [2012], who find persistent exposures of hedge funds to downside risk. Thus, it could be that (4) is not the right process for the hedge fund value and that a better model for the after management fee hedge fund value is given by

$$
\begin{aligned}
W_h(T) = {} & [W_h(0)(1 - f_h)^T] \\
& \times \exp\Big\{\big[r + (1+\pi)(\eta\sigma_{hi} + \alpha_h) \\
& \qquad - \frac{1}{2}(1+\pi)^2(\sigma_{hi}^2 + \tilde{\sigma}_{hh}^2)\big]T \\
& \qquad + (1+\pi)[\sigma_{hi}B_i(T) + \tilde{\sigma}_{hh}B_h(T)]\Big\} \\
& \times [1 + u(1+\pi)]^{N(T) - \tilde{\lambda}T}
\end{aligned}
\tag{10}
$$

Exhibit 7

Difference in Certainty Equivalents of Hedge Fund and Mutual Fund as a Function of Mutual Fund's Idiosyncratic Volatility (σ_{aa}) and Mutual Fund Alpha (α_a)

Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\sigma_{ai} = 0.2$, $f_h = 0.015$, $f_p = 0.2$, $\pi = 1$, $\alpha_h = 0.03$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, $L = 1$, and $\varepsilon = 0.02$.



where $\pi < -1 + 1/|u|$ and, thus, leverage is bounded, u is the jump size and it satisfies $u \in \left(-\sqrt{\sigma_{hh}^2/\lambda}, \sqrt{\sigma_{hh}^2/\lambda}\right)$, $N(t)$ is a Poisson process with intensity λ, $\tilde{\lambda}$ is a parameter that gives $E\left[[1+u(1+\pi)]^{N(T)-\tilde{\lambda}T}\right] = 1$ and it is given by

$$\tilde{\lambda} = \frac{Tu(1+\pi)}{T\log(1+u(1+\pi))}\lambda$$

and volatility parameter $\tilde{\sigma}_{hh}$ is given by

$$\tilde{\sigma}_{hh} = \sqrt{\sigma_{hh}^2 - u^2\lambda}$$

which means the variance of the hedge fund is independent of the jump risk. Note that condition $\pi < -1 + 1/|u|$ guarantees that $W_h(T) > 0$ and condition $u \in \left(-\sqrt{\sigma_{hh}^2/\lambda}, \sqrt{\sigma_{hh}^2/\lambda}\right)$ guarantees that volatility $\tilde{\sigma}_{hh}$ exits. Thus, (10) introduces a jump risk, and the model parameters are selected in such a way that the mean and the variance are independent of the jump risk. That is, (4) and (10) have the same first two moments, and here we analyze the effect of the higher-order moments on the hedge fund allocation. Exhibit 8 illustrates the situation.

As can be seen, when u is negative the jump risk raises the skewness and slightly decreases the kurtosis of the hedge fund value. That is, as expected, the distribution with the jump risk has a longer lower tail and a shorter upper tail.[11] Therefore, we expect that in this case our jump risk penalizes the hedge fund investor. Under the parameters of Exhibit 8, Panel B, the probability of zero jumps during a year is about 74%, and thus, one or more jumps is about 26%. In case of jump, the hedge fund value falls by 15%. Next we analyze the impact of the jump risk on the wealth allocation to the active mutual fund and the hedge fund.

Exhibit 9, Panel A, corresponds to Exhibit 2 and shows the difference in certainty equivalent between the hedge fund and the mutual fund; zero denotes indifference between the two options, and 0.01 (or 0.03) indicates that the investor prefers the hedge fund with a margin of 1% (or 3%) of his wealth per year. As can be seen, due to the jump risk in the hedge fund value, the investor is indifferent between the hedge fund and the mutual fund at a slightly higher hedge fund gross alpha. The average difference between the zero contours with and without the jump risk over the leverage levels between 0 and 4 is 0.003. That is, if the skewness falls by 1.11 (from 0.62 to −0.49, Exhibit 8), then on average (over all the leverage levels) the hedge fund should raise the gross alpha by 0.3% to keep the investor indifferent between the mutual fund and the hedge fund. The same average alpha differences for 0.01 and 0.03 contours are 0.5% and 0.8%, respectively. Thus, the effect of the jump risk rises in the hedge fund alpha and, by Exhibit 9, Panel A, and (10), also in the hedge fund leverage, because then both the expected hedge fund value and the jump risk are higher. By the risk aversion of the investor (Equation (7)), this makes the investor more reluctant to invest in the hedge fund.

Exhibit 9, Panel B, shows the difference in certainty equivalent between the hedge fund and the mutual fund with respect to hedge fund alpha and hedge fund skewness due to the jump risk. As we can see, the skewness has only a small impact on the portfolio allocation between the hedge fund and the mutual fund: the average slopes of the 0, 0.02, 0.04, and 0.06 contours are −0.002, −0.005, −0.006, and −0.008, respectively. Thus, for instance, if the skewness falls by one then the hedge fund gross alpha has to rise by 0.2% to keep the investor indifferent between the active mutual fund and the hedge fund (the zero contour in Exhibit 9, Panel B);

Exhibit 8

Hedge Fund Histogram with and without Jumps

The line represents the best fit normal distribution. Model parameters: $W_h(0) = 1$, $r = 0.03$, $\eta = 0.25$, $\alpha_h = 0.03$, $f_h = 0.015$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $T = 1$, $L = 1$, $\pi = 1$, $\lambda = 0.3$, $u = -0.15$, $\tilde{\sigma}_{hh} = 0.06$, and $\tilde{\lambda} = 0.25$. Number of simulation runs: 30,000.

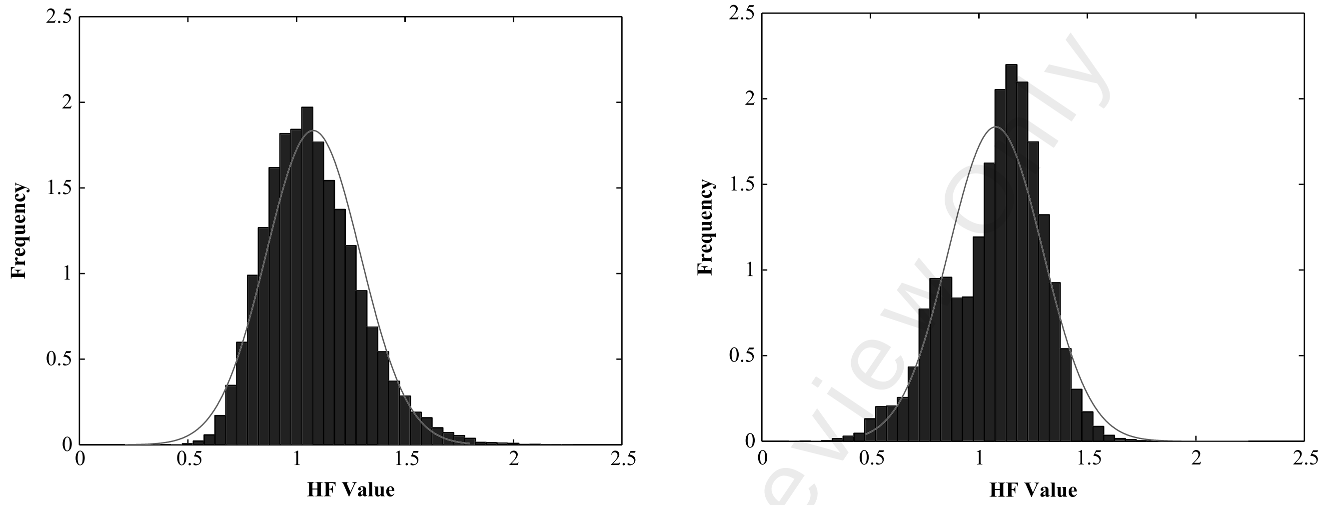
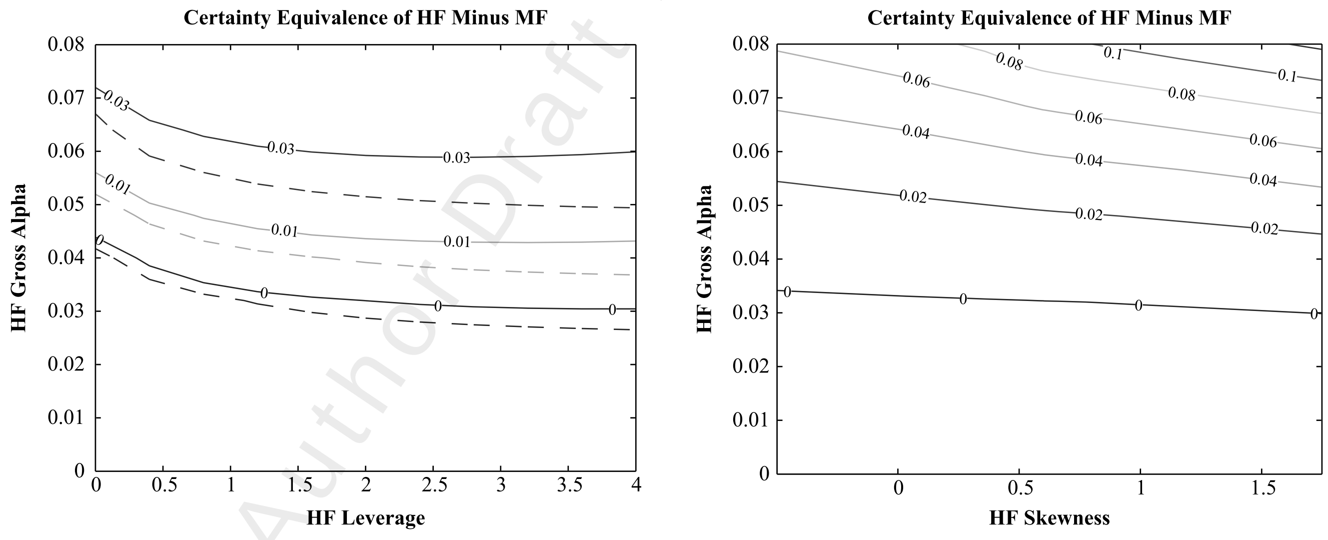


Exhibit 9

Difference in Certainty Equivalent Between Hedge Fund and Mutual Fund with Jump Risk

Model parameters: $r = 0.03$, $\eta = 0.25$, $\sigma_i = 0.20$, $f_a = 0.01$, $\alpha_a = 0.02$, $\sigma_{ai} = 0.2$, $\sigma_{aa} = 0.06$, $f_h = 0.015$, $f_p = 0.2$, $\sigma_{hi} = 0$, $\sigma_{hh} = 0.1$, $\gamma = 2$, $T = 1$, $L = 1$, $\lambda = 0.3$, and $\varepsilon = 0.02$.



and if the hedge fund has a 4% advantage and if the skewness falls by one, then the hedge fund alpha needs to rise by 0.6% for the hedge fund to keep that advantage (the 0.04 contour in Exhibit 9, Panel B). In this sense, our earlier results in this section are quite robust.

CONCLUSIONS

When an active equity manager with some moderate level of skill decides to run either a mutual fund or a hedge fund and settles on a typical fee structure for each type of fund, on the surface it seems that the mutual fund would deliver more value to investors because the fees appear to be lower. However, the two investment vehicles differ in many ways: besides the obvious difference in the level and structure of fees, hedge funds may have little or no market risk; instead they can use leverage to increase the size of their active bets. Which investment vehicle would be better for investors overall?

To answer this question, we build a simple model to compare a mutual fund with a hedge fund from the point of view of an investor allocating to either type of active fund as well as a market index fund and cash. We find that for a moderate level of skill, where the active manager's annual information ratio is about 0.3 before fees and expenses, both investment vehicles produce about the same expected utility to the investor, implying that the investor should be approximately indifferent between the two.

The investor benefits from hedge fund leverage since the management fee is paid only on the money invested in the fund, not on the levered gross positions. If the fund is more levered, the investor can effectively get the same exposure with a smaller investment in the fund. This offsets the higher hedge fund fees: the management fee and the incentive fee that is paid on positive profits. Furthermore, mutual funds generally offer too little exposure to their active bets and too much exposure to the broad market, whereas a market-neutral hedge fund position is easy to combine optimally with a passive position in the broad market index.

However, indifference between the hedge fund and mutual fund depends on parameter values that vary across funds. For example, our model shows that high-alpha fund managers offer more value to their clients if they work at hedge funds, because the additional leverage can really boost the alphas of the investors. Hedge funds with a higher incentive fee also face a much higher hurdle: our model shows that a 20% increase in the incentive fee should be compensated by about a 1% increase in the unlevered alpha. Some of these parameters are easily observable, such as fees, but others, such as alphas, are notoriously difficult to estimate, so for this reason we estimate the trade-off for a range of plausible values instead. Further, we show that our results are quite robust with respect to skewness of the hedge fund returns.

The objective of our article is to build a conceptual framework for comparing the two types of funds and not take a strong stance on specific inputs such as fund manager alphas. While this type of framework is a requirement for any rational comparison, a better empirical determination of hedge fund manager alphas is the next key step for researchers or investors interested in pushing this analysis further.

ENDNOTES

Acknowledgements: We have benefited from comments at Aalto University, Norwegian University of Science and Technology, University of Michigan, INFORMS Annual Meeting, and more specifically from Stein-Erik Fleten, Matti Keloharju, Samuli Knupfer, Mikko Leppamaki, Peter Molnar, Romesh Saigal, Matti Suominen, Sami Torstila, and Tuomo Vuolteenaho. We are also grateful to Vinay Benny and Zhichen Zhao for research assistance.

[1]See, e.g., "Hedge Funds: Fees Down? Close Shop," available at http://www.businessweek.com/stories/2005-08-07/hedge-funds-fees-down-close-shop.

[2]We have also a technical condition: Our utility function in (7) assumed strictly positive wealth.

[3]We coded the algorithm in Matlab and by utilizing the fmincon optimization function there.

[4]The average gross leverage across all hedge funds is 2.13 in Ang et al. [2011], which would correspond to $\pi = 1.13$ in our article. Note that the definition of leverage used by Ang et al. [2011] and most practitioners is more like a "leverage factor": when leverage equals one according to their definition, a fund would invest 100% of its own capital but would not actually borrow money. In contrast, we follow a more literal definition where a leverage of one (i.e., 100%) implies $200 worth of positions for $100 of capital, and only a leverage of zero means no borrowing (and negative leverage would mean lending money at the cash rate).

[5]According to Petajisto [2010], stock picker funds have alpha of 2.6% before fees with 8.5% tracking error, which gives an information ratio of 0.31. Kosowski et al. [2007] report an average hedge fund alpha of 5.0% net of all expenses and transaction costs, which implies a slightly higher information ratio.

[6]In reality, one could plausibly argue that hedge fund managers should perform slightly better because they have the flexibility to pursue a broader range of active strategies, and their performance-based compensation structure may also be more appealing to a manager who actually has skill. However, our question in this article is about the impact of the fund structure on the net returns to investors, so we actually want to assume the same fundamental skill for both types of managers.

[7]We get this from the first-order condition: $(1/\gamma) \cdot (\eta\sigma_i/\sigma_i^2) = (1/2) \cdot 0.05/0.2^2 = 0.625 = 62.5\%$.

[8] Since some hedge fund investors (such as banks, insurance companies, and high-net-worth individuals) can use leverage, our model allows the investor to use leverage (for more information on the source of hedge fund investors, see, e.g., Financial Services Authority [2012]). This modeling choice is not critical, but it makes our results more robust as they are not driven by a borrowing constraint on the investor.

[9] If the hedge fund alpha was higher, then the indifference leverage level would be lower. For instance, for 4% hedge fund alpha, the leverage level is about zero.

[10] For example, both institutional pension fund investors and individually managed pension fund accounts generally cannot or will not go short.

[11] For instance, Financial Services Authority [2011] reports six-month hedge fund returns that range between –15% and 10% and that also have a negative skewness (time period: April–September 2010). We do not model explicitly hedge fund liquidation (for an example of liquidation see, e.g., *Wall Street Journal*, March 29, 2002, "Several Kenneth Lipper Hedge Funds Are Being Liquidated After Big Losses," available at http://online.wsj.com/news/articles/SB1017355983517177960). According to Darolles et al. [2013], the global annual liquidation rate for hedge funds between 1994 and 2003 was around 8%–9%, which corresponds to a median lifetime of 6–7 years. Brown et al. [2001] report that negative returns over one-year and two-year horizons increase the likelihood of liquidation. Further, ter Horst and Verbeek [2007] analyze hedge fund returns during 1994–2000 and find that the average quarterly return of hedge funds that were liquidated in their dataset is 0.50%, while it is 3.59% for funds that survived until 2000 (the corresponding average quarterly net flows are 2.49% and 9.07%).

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